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THE MAXIMUM-MINIMUM SHIFT METHOD FOR MEASURING COMPLEX DIELECTRIC CONSTANTS AND PERMEABILITIES

By

Ronald King

December 15, 1953

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The Maximum-Minimum Shift Method for Measuring Complex Dielectric Constants and Permeabilities

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Ronold King

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Abstract

An absolute method for measuring dielectric constants of solids and liquids which is described in the literature[1,2] is generalized to permit the determination of both dielectric constant and permeability of a moderately low-loss solid or fluid medium. The method is absolute in the sense that only measurements of length are required to determine ϵ_r and μ_r. A special feature is the fact that ϵ_r and μ_r are each determined under conditions of maximum sensitivity. The determination of losses involving complex dielectric constants and permeabilities is also described.

Introduction

The maximum-minimum shift method is a simple, direct procedure for the simultaneous determination of both the relative dielectric constant, ϵ_r, and the relative permeability, μ_r, of a slab of material of convenient thickness inserted in a coaxial or other transmission line. In its original form[1,2] it was described only for measuring the relative dielectric constant. However, it is extended readily to include the simultaneous determination of the relative permeability of moderately low-loss materials. These losses also may be determined.

The fundamental principle of the method is very simple. In effect, it involves merely the successive measurement of the impedance of a section of transmission line when immersed in the material under test when terminated in an open and a short circuit. Since the sample to be used may be chosen to be symmetrical, it is convenient to make use of the symmetrical and anti-symmetrical combinations involving respectively, an open circuit and a short circuit in the plane through the center of the slab.

Although it may appear to be experimentally simpler to arrange successively a short circuit and an open circuit at the back surface of a slab of material rather than at its center, it turns out that the very process of locating the equivalent of a short circuit and an open circuit at the center of the sample constitutes the essential data required for the determination of both ϵ_r and μ_r.

The principle involved in locating open and short circuits at the center of a slab of material of thickness d depends upon the fact that the effect of the slab in modifying the condition of resonance of a section of transmission line is extreme -- either maximum or minimum -- when the slab is symmetrically or antisymmetrically located with respect to the current and voltage distribution patterns. A location in which a voltage maximum and a current null are at the center of the slab is symmetrical with respect to the voltage and antisymmetrical with respect to the current. It is equivalent to an open circuit at the center. A location in which a voltage null and a current maximum are at the center of the slab is antisymmetrical with respect to the voltage, symmetrical with respect to the current. It is equivalent to a short circuit at the center. Actually, completely symmetrical distributions of current and voltage (in which current or voltage nulls rather than minima occur at the center of the slab) are achieved only if the slab is itself exactly at the center of a resonant symmetrical section of line that is driven by identical generators loosely coupled at both ends. If the generators are in phase, there is a voltage null at the center of the slab; if they are 180° out of phase, there is a current null at the center of the slab. In practice, the slab may be placed with its center at a voltage or current maximum with sections of low-loss line on each side. Only one of these sections need be driven by a loosely coupled generator if the material in the slab is not highly dissipative so that the circuit as a whole has a moderately high Q as indicated by the sharpness of the resonance curves. The detector preferably is coupled to the same section as the generator.

Mathematical Formulation

The first step in deriving the tangent relation upon which the maximum-

minimum shift method depends is to compare the input admittance of two sections of highly conducting transmission line. Of these the first (Fig. 1a) has only air (vacuum) as the dielectric from the arbitrarily located input terminals at $z' = 0$ to the reactive termination with admittance $\underline{Y}_T = jB_T$ at $z' = s'$. The second section of line (Fig. 1b) is immersed in a medium with complex dielectric factor $\underline{\xi}_1$ and complex permeability $\underline{\mu}_1$ from $z' = 0$ to $z' = d$, and in air from $z' = d$ to $z' = d + s$, where it is terminated in $\underline{Y}_T \doteq jB_T$. The complex material parameters of the homogeneous isotropic medium are

$$\underline{\epsilon} = \epsilon' - j\epsilon'' \; ; \; \underline{\mu} = \mu' - j\mu''; \; \underline{\sigma} = \sigma' - j\sigma'' \tag{1a}$$

These occur in the following forms:

$$\underline{\xi} \equiv \epsilon_e - \frac{j\sigma_e}{\omega} = \epsilon_e(1-jh_e) \; ; \; h_e \equiv \frac{\sigma_e}{\omega\epsilon_e} = \frac{\sigma' + \omega\epsilon''}{\omega\epsilon' - \sigma''} \tag{1b}$$

$$\underline{\mu} = \mu'(1-jh_m) \; ; \; h_m \equiv \frac{\mu''}{\mu'} \tag{1c}$$

The relative dielectric constants and permeabilities are obtained from the absolute values in (1), (2), and (3) by dividing by ϵ_o and μ_o, respectively. Thus,

$$\underline{\epsilon} = \epsilon_o \underline{\epsilon}_r \; ; \; \epsilon' = \epsilon_o \epsilon'_r \; ; \; \epsilon'' = \epsilon_o \epsilon''_r \tag{1d}$$

$$\underline{\mu} = \mu_o \underline{\mu}_r \; ; \; \mu' = \mu_o \mu'_r \; ; \; \mu'' = \mu_o \mu''_r \tag{1e}$$

The effective conductivity σ_e includes ohmic losses arising from actual conduction in σ' and from time-lags in polarization in $\omega\epsilon''$. Time-lags in magnetization contribute the term in μ''. The conditions restricting the medium to relatively low losses are

$$h_e^2 \ll 1 \; ; \; h_m^2 \ll 1 \tag{1f}$$

In the following the subscript e, denoting an effective value, is omitted from ϵ and σ with the understanding its presence is required if $\omega\epsilon''$ contribute significantly to σ_e and σ'' contributes significantly to $\omega\epsilon_e$ in (1b). If the medium is a liquid, retaining walls are required. These may be ignored if made of a material like Polyfoam that has a relative dielectric constant and a relative permeability differing negligibly from one. If they are made

of a solid dielectric, they may be sufficiently thin to permit their analytical representation as small lumped admittances $\underline{Y}_w = jB_w = -j\omega C_w$ at $z' = 0$ and $z' = d$ (Fig. 1c). In the following the more general problem involving a liquid enclosed in thin, solid retaining walls is formulated since the results are specialized readily to the simpler and more important cases involving liquids with Polyfoam walls or a solid dielectric with no additional walls, by setting $B_w = 0$.

The input admittance $\underline{Y}'$ for the section of line in Fig. 1a is

$$\underline{Y}' = G' + jB' = \underline{Y}_c \coth(\underline{\gamma} s' + \underline{\theta}_T') \doteq -jG_c \cot(\beta s' + \Phi_T') \tag{2}$$

The characteristic admittance of the line is $\underline{Y}_c \doteq G_c = 1/R_c$; the propagation constant is $\underline{\gamma} = \alpha + j\beta$. $\theta_T' = \coth^{-1}(\underline{Y}_T/\underline{Y}_c) = \rho_T + j\Phi_T' \doteq j\Phi_T'$ is the terminal function of $\underline{Y}_T$; for an ideal short circuit, $\underline{\theta}_T' = 0$. The values following the approximately equal sign in (1) and the related definitions apply only if the attenuation of the line is neglected and the termination is a pure reactance.

The input admittance $\underline{Y}_2$ of the section of line of length s in Fig. 1c in parallel with the lumped admittance $\underline{Y}_w \doteq jB_w$ of the right hand retaining wall is

$$\underline{Y}_2 = G_2 + jB_2 = \underline{Y}_w + \underline{Y}_c \coth(\underline{\gamma} s + \theta_T') \doteq j[B_w - G_c \cot(\beta s + \Phi_T')] \tag{3a}$$

The input admittance $\underline{Y}$ of the entire line in Fig. 1c is

$$\underline{Y} = G + jB = \underline{Y}_w + \underline{Y}_{c1}\left[\frac{\underline{Y}_2 \coth \underline{\gamma}_1 d + \underline{Y}_{c1}}{\underline{Y}_2 + \underline{Y}_{c1} \coth \underline{\gamma}_1 d}\right] \tag{3b}$$

where $\underline{Y}_{c1} = G_{c1}(1 + j\phi_{c1}) \doteq G_c$ is the characteristic admittance and $\underline{\gamma}_1 = \alpha_1 + j\beta_1$, the propagation constant of the line when immersed in the slab of material medium between $z = 0$ and $z = d$.

The fundamental step in the derivation of the desired equation is to require the lengths s' and s to be so related that the input susceptances B and B' are equal. Thus,

$$B = B' \quad \text{or} \quad \text{Im}\,\underline{Y} = \text{Im}\,\underline{Y}' \tag{4}$$

with the properties of the dielectric material as represented by $\underline{\gamma}_1$ and $\underline{Z}_{c1}$ and its thickness d arbitrary.

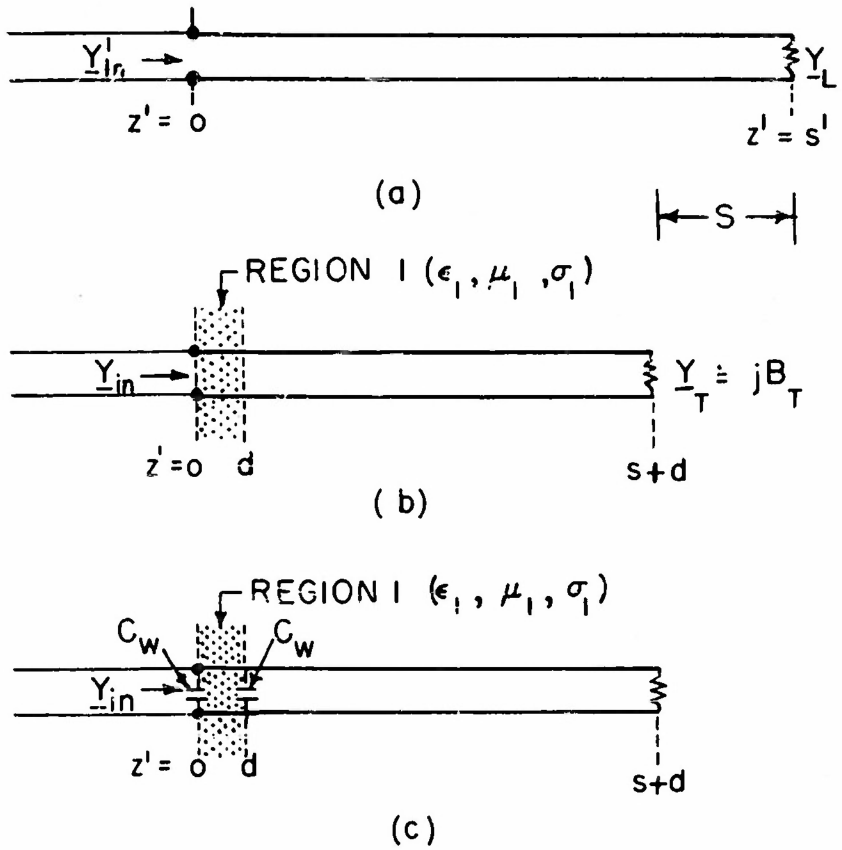

FIG. I SECTIONS OF LINE WITH (a) NO DIELECTRIC SLAB, (b) SOLID DIELECTRIC SLAB, (c) FLUID WITH SOLID RETAINING WALLS

It is readily verified that, using (2) and (3b) with (3a), (4) may be transformed into the following general equation:

$$\mathrm{Re}\left\{\underline{C}_1[\coth(\underline{\gamma}s' + \underline{\theta}'_T) - \coth(\underline{\gamma}s + \underline{\theta}'_T)] + \coth(\underline{\gamma}s' + \underline{\theta}'_T)\coth(\underline{\gamma}s + \underline{\theta}'_T) - \underline{C}_2\right\} = 0 \tag{5}$$

where

$$\underline{C}_1 = C_{1r} + jC_{1i} = \underline{r}_c \coth \underline{\gamma}_1 d + \underline{Y}_w \underline{Z}_c \tag{6a}$$

$$\underline{C}_2 = C_{2r} + jC_{2i} = \underline{r}_c^2 + 2\underline{r}_c \underline{Y}_w \underline{Z}_c \coth \underline{\gamma}_1 d + \underline{Y}_w^2 \underline{Z}_c^2 \tag{6b}$$

with

$$\underline{r}_c = \frac{\underline{Z}_c}{\underline{Z}_{c1}} \tag{7}$$

With these definitions of $\underline{C}_1$ and $\underline{C}_2$, it is the real part of (5) that is derived from the susceptance. Equation (5) expresses the relationship between all values of s' and s for which the input susceptances of the two sections (the one of length s' in air; the other of length d in the dielectric or magnetic medium and length s in air) are equal. In the complete absence of the material medium ($d = 0$, $B_w = 0$) the input admittances are equal when $s' = s_1$. If a dielectric or magnetic medium is present, d+s is taken to be less than s'.

The greatest effect on the input susceptance B_{in} is produced by the material medium when it is so situated that the values of s' and s that satisfy (5) are such that s'-s is a maximum. Evidently, with this combination of s' and s the circuit reaches its greatest sensitivity to the reactive effect of a dielectric or magnetic sample so that it represents the optimum condition for the precise measurement of ϵ_r or μ_r.

The particular forms of the equation (5) for which s'-s has its extreme values are obtained by setting the derivative of s'-s with respect to s' equal to zero or, what is equivalent, by setting

$$\frac{ds}{ds'} = \frac{d(\gamma s)}{d(\gamma s')} = 1 \tag{8}$$

Using standard formulas, let (5) be transformed into the following equivalent expression:

$$\text{Re}\left\{-\underline{C}_1 \sinh\underline{\gamma}(s'-s)+\left(\frac{1+\underline{C}_2}{2}\right)\cosh\underline{\gamma}(s'-s)+\left(\frac{1-\underline{C}_2}{2}\right)\cosh[\underline{\gamma}(s'+s)+2\underline{\Theta}'_T]\right\} = 0 \quad (9)$$

If (9) is differentiated with respect to s' and (8) is imposed, the following condition is obtained:

$$(\underline{C}_2 - 1)\sinh[\underline{\gamma}(s'+s) + 2\underline{\Theta}'_T] = 0 \quad (10)$$

Since $\underline{C}_2$ is not equal to unity in general, (10) is equivalent to the following:

$$\sinh[\alpha(s'+s)+2\rho_T]\cos[\beta(s'+s)+2\Phi'_T] + j\cosh[\alpha(s'+s)+2\rho_T]\sin[\beta(s'+s)+2\Phi'_T] = 0 \quad (11)$$

where only the imaginary part is relevant for the condition (4). This part of (11) is satisfied when

$$\beta(s'+s) + 2\Phi'_T = k\pi, \quad k = 0,1,2,\ldots \quad (12)$$

Using (12) in (9), the following equation is obtained for the maximum and minimum values of (s'-s) (indicated by the subscript m):

$$C_{1i}\sin\beta(s'-s)_m + \tfrac{1}{2}(1+C_{2r})\cos\beta(s'-s)_m \pm \tfrac{1}{2}(1-C_{2r}) = 0 \quad (13a)$$

where the upper sign is for k even, the lower sign for k odd in (12), and where C_{1i} and C_{2r} are the imaginary part of $\underline{C}_1$ and the real part of $\underline{C}_2$, respectively. In deriving (13a) it is assumed that the following inequalities are good approximations:

$$|C_{1i}| >> |C_{2i}\alpha(s'-s)| \quad (13b)$$

$$[\alpha(s'+s) + \rho_T]^2 << 1 \quad (13c)$$

The first of these conditions implies that the distortion factor ϕ_{c1} in the characteristic impedance $\underline{Z}_{c1} = R_{c1}(1-j\phi_{c1})$ of the dielectric sample is quite small, the second that the line outside the dielectric medium has low losses. Conditions essentially equivalent to these are imposed in (18).

Equation (13a) is transformed readily into the following two equations:

$$\cot^2\Delta + 2C_{1i}\cot\Delta - C_{2r} = 0 \;; \quad k \text{ even in } (12) \quad (14a)$$

$$\tan^2\Delta - 2C_{1i}\tan\Delta - C_{2r} = 0 \quad ; \quad k \text{ odd in (12)} \tag{14b}$$

where the notation

$$\Delta \equiv \frac{1}{2}\beta(s'-s)_m = \frac{1}{2}\beta(d + S_m) \tag{15}$$

is introduced. In (15) S_m is the maximum or minimum shift in the position of the termination when adjusted for resonance successively without and with the material medium. The shift S is shown in Fig. 1. The solutions of (14a) and (14b) are

$$\cot\Delta = - C_{1i} \pm \sqrt{C_{1i}^2 + C_{2r}} \tag{16}$$

$$\tan\Delta = C_{1i} \pm \sqrt{C_{1i}^2 + C_{2}} \tag{17}$$

where, for $2\Delta = \beta(s'-s)$ positive and less than π, only the upper signs are relevant. The complex constants $\underline{C}_1 = C_{1r} + jC_{1i}$ and $\underline{C}_2 = C_{2r} + jC_{2i}$ are defined in (6a,b). Although the real and imaginary parts of $\underline{C}_1$ and $\underline{C}_2$ are separable in general without restricting the properties of the material in the slab under investigation, resonance curves are sharp only for materials that are not very good conductors. Accordingly, it is convenient to obtain the simpler formulas which apply to samples of moderately low effective conductivity. This is in agreement with the conditions (1f). Therefore, let the following restrictions be imposed on the propagation constant $\underline{\gamma}_1 = \alpha_1 + j\beta_1$ and the characteristic impedance $\underline{Z}_{c1} = R_{c1}(1-j\phi_{c1})$:

$$(\alpha_1 d)^2 << 1 \quad ; \quad \left(\frac{\alpha_1}{\beta_1}\right)^2 << 1 \quad ; \quad \phi_{c1}^2 << 1 , \tag{18}$$

Subject to these conditions,

$$\coth \underline{\gamma}_1 d \doteq -j\cot \beta_1 d + \alpha_1 d\csc^2\beta_1 d \tag{19}$$

provided the additional requirement,

$$\tan^2\beta_1 d >> \alpha_1^2 d^2 \tag{20}$$

is satisfied. With (18) and (7) it follows that

$$\underline{r}_c = \frac{R_c(1-j\phi_c)}{R_{c1}(1-j\phi_{c1})} \doteq r_c(1+j\phi_r) \; ; \; r_c = \frac{R_c}{R_{c1}} = \sqrt{\frac{\epsilon_r}{\mu_r}} \; ; \; \phi_r \doteq \phi_{c1} - \phi_c \tag{21}$$

where ϵ_r and μ_r are the relative dielectric constant and permeability of the material medium. Note that

$$\beta_1 = n\beta \quad ; \quad n = \sqrt{\epsilon_r \mu_r} \tag{22}$$

where n is the index of refraction. In a nonmagnetic dielectric material $\mu_r = 1$, $r_c = n = \sqrt{\epsilon_r}$; in a nondielectric magnetic material $\epsilon_r = 1$, $r_c = 1/\sqrt{\mu_r} = 1/n$.

With (18) through (22) it follows from (7a,b) that

$$C_{1i} = B_w R_c - r_c \cot n\beta d \tag{23}$$

$$C_{2r} = r_c^2 + 2r_c B_w R_c \cot n\beta d - B_w^2 R_c^2 \tag{24}$$

so that

$$-C_{1i} + \sqrt{C_{1i}^2 + C_{2r}} = -B_w R_c + r_c \cot \tfrac{1}{2} n\beta d \tag{25}$$

$$C_{1i} + \sqrt{C_{1i}^2 + C_{2r}} = B_w R_c + r_c \tan \tfrac{1}{2} n\beta d \tag{26}$$

If (25) and (26) are substituted in (16) and (17) these may be expressed as follows.

$$\cot \Delta_i + B_w R_c = r_c \cot \tfrac{1}{2} n\beta d \; ; \quad k \text{ even in (12)} \tag{27a}$$

$$\tan \Delta_v - B_w R_c = r_c \tan \tfrac{1}{2} n\beta d \; ; \quad k \text{ odd in (12)} \tag{27b}$$

It is readily verified that the condition (12), $\beta(s'+s) + 2\Phi_T' = k\pi$, $k = 0,1,2,\ldots$, assures that the extreme values $\Delta_i = \frac{1}{2}\beta(s-s')_{mi}$ and $\Delta_v = \frac{1}{2}\beta(s-s')_{mv}$, occur when the current and voltage distribution patterns are symmetrical with respect to the center of the slab. With (12) the part of the susceptance B_2 in (3a) due to the line is given by

$$B_{2in} \equiv B_2 - B_w = -G_c \cot[k\pi-(\beta s' + \Phi'_T)] = G_c \cot(\beta s' + \Phi'_T) \quad (28a)$$

Since the susceptance B' in (2) is equal to the susceptance B in (3b), and since B_{2in} in (28a) is the negative of B' in (2), it follows that when located for extreme shift,

$$B_{2in} = -B \quad (28b)$$

However, since the entire circuit is adjusted for resonance, the susceptance B looking into the slab must be the negative of the susceptance at the same points but looking away from the slab back into the line.

$$B_{2in} = -B = B_L \quad (28c)$$

That is, the susceptances looking into the line in both directions from the edges of the dielectric slab are the <u>same</u>. This is possible only when the current and voltage distributions are symmetrical with respect to the center of the slab. In particular, the extreme value Δ_i defined in (27) always occurs when the largest number of current maxima consistent with the electrical thickness $n\beta d$ of the sample are contained within it. When $n\beta d$ is less than π, this means a current maximum at the center of the slab. (When $n\beta d$ is between π and 2π, it means voltage maximum at the center with two symmetrically placed current maxima within the slab.) Alternatively, the extreme value Δ_v defined in (28) occurs when the largest number of voltage or charge maxima are contained within the sample. For $n\beta d$ less than π, this means a voltage maximum at the center of the slab. The question as to which of the two extreme values Δ_i and Δ_v is a maximum and which a minimum depends on the relative magnitudes of ϵ_r and μ_r. If $\mu_r = 1$, $\epsilon_r > 1$, Δ_v is the maximum, Δ_i the minimum. If $\epsilon_r = 1$, $\mu_r > 1$, Δ_i is the maximum, Δ_v the minimum. If $\epsilon_r = \mu_r$ there is only one value of $s'-s$ for all positions of the slab so that Δ_v and Δ_i are equal.

Equations (27a) and (27b) may be solved for $r_c = \sqrt{\epsilon_r/\mu_r}$ and $n = \sqrt{\mu_r \epsilon_r}$. The product of (27a) and (27b) is

$$\sqrt{\epsilon_r/\mu_r} = r_c = [\cot \Delta_i \tan \Delta_v + B_w R_c (\tan \Delta_i - \cot \Delta_v) - B_w^2 R_c^2]^{1/2} \quad (29a)$$

The ratio of (28) to (27) is

$$\sqrt{\mu_r \epsilon_r} = n = \frac{2}{\beta d} \tan^{-1} \left[\frac{\tan\Delta_i - B_w R_c}{\cot \Delta_i + B_w R_c} \right]^{1/2} \tag{29b}$$

If the sample includes no solid retaining walls, $B_w = 0$ and the following simpler expressions are obtained:

$$\sqrt{\epsilon_r / \mu_r} = r_c = (\cot \Delta_i \tan \Delta_v)^{1/2} \tag{30a}$$

$$\sqrt{\epsilon_r \mu_r} = n = \frac{2}{\beta d} \tan^{-1} (\tan \Delta_i \tan \Delta_v)^{1/2} \tag{30b}$$

It follows that

$$\epsilon_r = r_c n \doteq \frac{2}{\beta d} [(\cot \Delta_i \tan \Delta_v)]^{1/2} \tan^{-1} (\tan \Delta_i \tan \Delta_v)^{1/2} \tag{31a}$$

$$\mu_r = n/r_c = \frac{2 \tan^{-1} (\tan \Delta_i \tan \Delta_v)^{1/2}}{\beta d(\cot \Delta_i \tan \Delta_v)} \tag{31b}$$

If the two extreme values $(s'-s)_i$ and $(s'-s)_v$ in Δ_i and Δ_v are determined experimentally, the relative dielectric constant ϵ_r and the relative permeability μ_r of the sample may be determined from (29a,b) or from (31a,b). The only other quantities required are the thickness d of the sample and the wavelength λ in $\beta = 2\pi/\lambda$ for the line in air. Thus, since only four length measurements are involved, an absolute method for the determination of ϵ_r and μ_r is available.

If a liquid material is contained between solid retaining walls for which B_w is not zero, $B_w R_c$ may be determined experimentally using (27) or (28) with the cell empty. In this case $r_c = n = 1$, so that

$$B_w R_c = \tan \Delta_{ve} - \tan \tfrac{1}{2}\beta d = \cot \tfrac{1}{2}\beta d - \cot \Delta_{ie} \tag{32a}$$

where $\Delta_{ve} = \frac{1}{2}\beta(s'-s)_{mv}$ and $\Delta_{ie} = \frac{1}{2}\beta(s'-s)_{mi}$ for the cell empty. Alternatively, the susceptance B_w of the walls may be eliminated by subtracting the equations for the empty cell from those for the full cell. For (28), for example, the result is

Experimental Procedure

In order to measure dielectric constants and permeabilities by the extreme-shift method, a sample of the material of thickness d must be moved along a transmission line that has a loosely coupled generator and a loosely coupled detector fixed near one end, and a movable reactive termination (e.g., a piston) $\underline{Y}_T = j\underline{B}_T$ at the other as shown in Fig. 2 with $Y_T = 0$. The first operation is to locate the position b' (Fig. 2a) of the reactive termination at which the circuit without dielectric is tuned to resonance as indicated by a maximum deflection of the detector.

The second operation is to move the slab of material (or the cell containing the liquid) from the point b' toward the detector step by step thus increasing the distance s' between b' and the left-hand surface of the dielectric. For each position of the dielectric the reactive termination is moved toward the dielectric to b where the circuit is again tuned to resonance as indicated by a maximum deflection of the detector. The distance between the termination at the resonant position b and the right side of the dielectric is s. As s' and s are increased step by step, but in general at different rates, a "shift curve" may be plotted of the difference s'-s as a function of the location along the line of the center of the slab. The origin of the linear scale along the line is arbitrary. Typical "shift curves" for water solutions of ethyl alcohol for which $\epsilon_r > 1$ with $\mu_r = 1$ are in Fig. 3.

As s' is increased by moving the sample toward the detector, a point is reached where the reactive termination must be moved away from rather than toward the dielectric in order to tune the circuit to resonance. At this point a further increase in s' results in a decrease in (s'-s) -- evidently the maximum value $(s'-s)_{mv}$ of (s'-s) has been reached. For a certain range beyond this maximum (s'-s) decreases as s' is increased. Then (s'-s) reaches a minimum $(s'-s)_{mi}$, and again starts increasing with continually increasing s'. As indicated in Fig. 3 the center of the dielectric slab is at a voltage minimum when (s'-s) is a minimum. If the reactive termination is a perfect short circuit (e.g., a piston), and the slab is electrically thin, the center of the dielectric is $\lambda/2$ from b' when (s'-s) is a minimum.

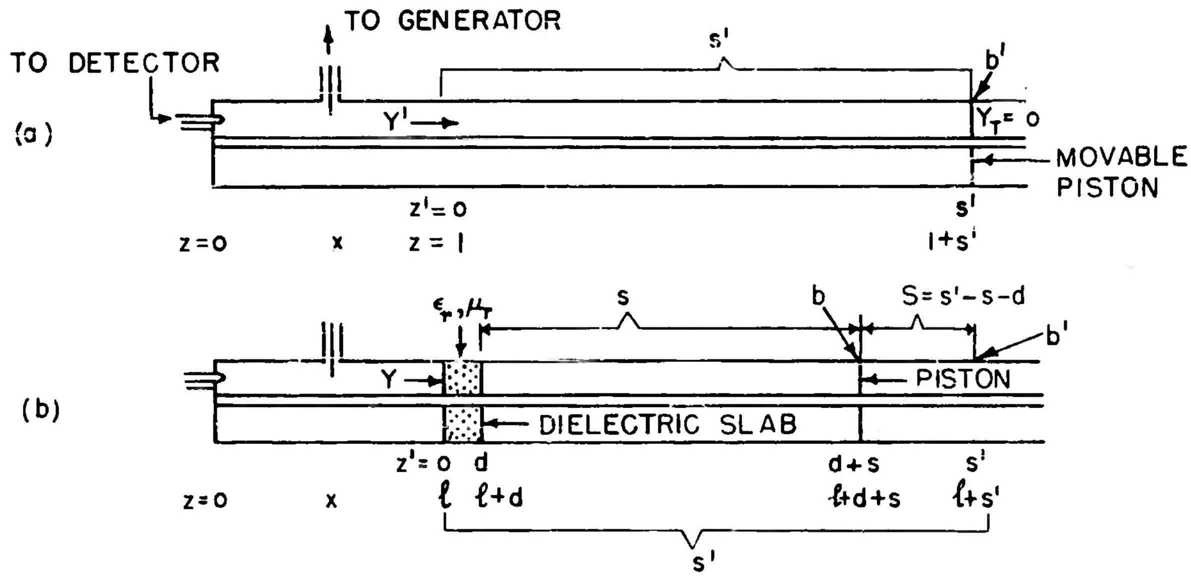

FIG. 2 (a) LOCATION OF PISTON AT b' FOR RESONANCE WITH NO DIELECTRIC

(b) LOCATION OF PISTON AT b FOR RESONANCE WITH DIELECTRIC AT $z = \ell$

In order to determine the dielectric constant of a material with $\mu_r = 1$, it is sufficient to measure the maximum value $(s'-s)_{mv}$ of $(s'-s)$; it is not necessary to plot a complete shift curve like those in Fig. 3. Several experimentally determined maximum values (without the rest of the associated shift curves) also are shown in Fig. 3. Note also that they all lie on the straight line of slope 2 as required by (35b); (in Fig. 3 s' increases from left to right).

With $(s'-s_2)_{mv}$ measured, and $\beta = 2\pi/\lambda$ known (or measured), $n = r_c = \sqrt{\epsilon_r}$ may be evaluated from (28) with $B_w = 0$ or from (34) if there are solid retaining walls.

The Size of the Sample

The mathematical theory assumes that the sample under test consists of a flat slab of thickness d with its parallel sides perpendicular to the axes of the conductors and completely filling the space between and around them. For use in a coaxial or shielded-pair line it consists of a disk that fits into the outer conductor or shield and has a hole or holes for the inner conductor or conductors. For use on an open-wire line the dielectric must ideally extend to infinity, although a relatively small properly shaped sample may be used if its relative dielectric constant or permeability is not too near one and a correction is made for the fraction of the field outside the sample.* In general, measurements are most convenient with a coaxial line.

In order to determine the most useful value for the thickness d of the sample, it is necessary to consider both the magnitude of the dielectric constant and permeability and the frequency at which it is to be measured. In Fig. 4 theoretical curves are shown of the index of refraction $n = \sqrt{\epsilon_r}$ as a function of the argument $\Delta_v = \frac{1}{2}\beta(s'-s)_{mv}$ for a range of values of $\frac{1}{2}\beta d$ as determined from the fundamental equation (28) with $\mu_r = 1$ and $B_w = 0$.

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See R. King, Rev. Sci. Instr 8, 201 (June, 1937).

$$\tan \Delta_v = n \tan \frac{1}{2} n\beta d \quad ; \quad n = \sqrt{\epsilon_r} \quad : \quad k \text{ odd in (12)} \qquad (36)$$

With the aid of these curves it is possible to estimate the thickness d of the sample required to produce an adequate maximum value of (s'-s) if the order of magnitude of the unknown dielectric constant is known as well as the frequency.

If the dielectric constant of a liquid is to be measured, a closed movable cell is required. Its parallel sides may be of Polyfoam or very thin solid dielectric; its inner and outer circular walls should be metal sleeves that slide over the inner and into the outer conductor of the coaxial line. By means of metal tubes of the same sizes as the sleeves, the entire section of line from the front of the dielectric sample to the reactive termination (piston) at b may be made to have constant inner and outer radii. The fact that these differ from the values between the detector and the front of the cell is immaterial, since only the distances s', s, and d occur in the final formula.

The determination of μ_r for materials with $\epsilon_r \doteq 1$ parallels the determination of ϵ_r for materials with $\mu_r = 1$. With $B_w = 0$ and $\epsilon_r = 1$ in (27) this becomes

$$\tan \Delta_i = n \tan \frac{1}{2} n\beta d \quad ; \quad n = \sqrt{\mu_r} \quad ; \quad k \text{ even in (12)} \qquad (37)$$

Since this is the same as (36) except for a differently defined n and different k in (12), the curves of Fig. 4 may be used.

Since $\Delta_v = \frac{1}{2} \beta(s'-s)_{mv}$ depends primarily upon ϵ_r and $\Delta_i = \frac{1}{2}\beta(s'-s)_{mi}$ upon μ_r, the curves of Fig. 4 are satisfactory for estimating the thickness d even in the general case when μ_r and ϵ_r both differ from unity. In general, $(s'-s)_v$ is the maximum, $(s'-s)_i$ the minimum shift when ϵ_r is greater than μ_r: $(s'-s)_i$ is the maximum and $(s'-s)_v$ the minimum when μ_r is greater than ϵ_r. As μ_r and ϵ_r approach each other, the maximum and minimum flatten until the shift curve is a straight line when $\mu_r = \epsilon_r$.

Measurement of Small Susceptances

The maximum shift method is a highly sensitive procedure for

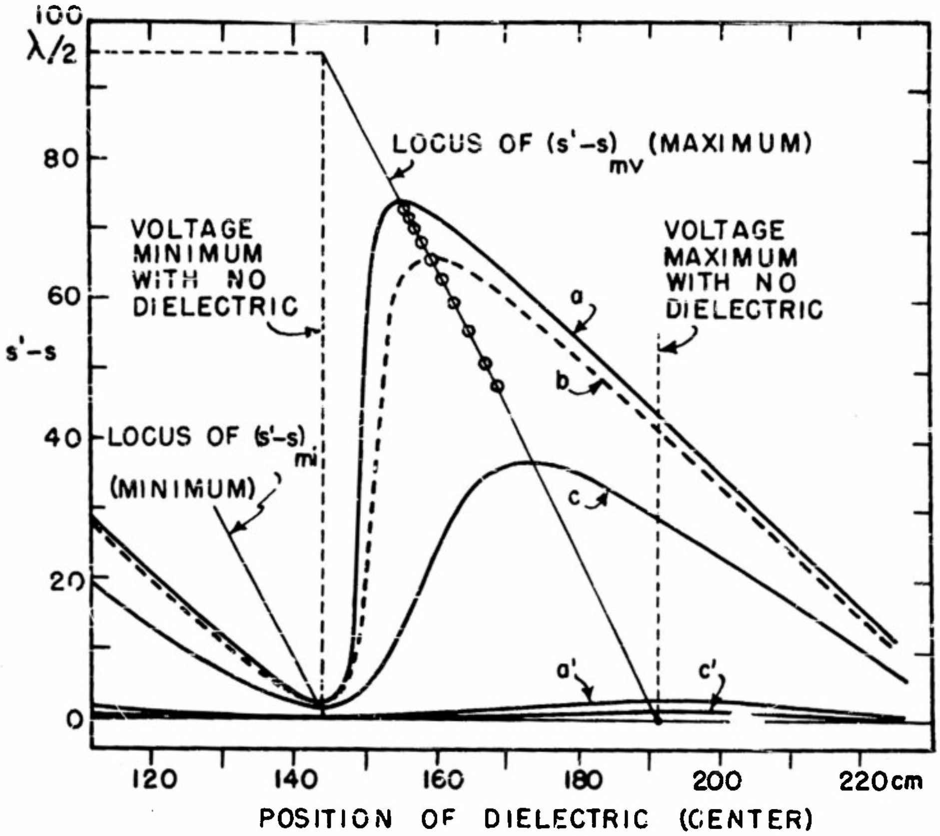

FIG. 3 SHIFT CURVES; a – DISTILLED WATER AT 22°.4 C IN CELL WITH d = 2.08 cm; a' – SAME CELL EMPTY; c – DISTILLED WATER AT 21°.7 C IN CELL WITH d = 0.52 cm; c' – SAME CELL EMPTY; LARGE CIRCLES ARE MAXIMA OF SHIFT CURVES FOR WATER SOLUTIONS OF ETHYL ALCOHOL; b IS ONE OF THESE CURVES COMPLETELY PLOTTED. (λ = 188.8 cm)

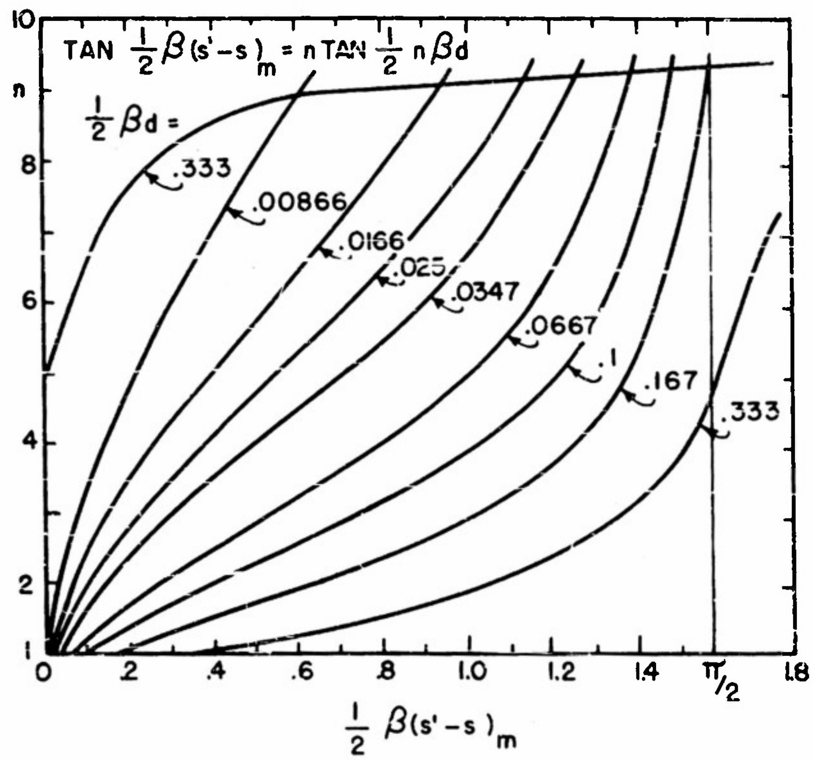

FIG. 4 CURVES OF n AS FUNCTION OF $\frac{1}{2}\beta(s'-s)_m$ AS DETERMINED FROM EQUATION (36) WITH $\frac{1}{2}\beta d$ AS PARAMETER

measuring small lumped susceptances. The appropriate formula is obtained directly from (28) by setting d = 0 and combining the two lumped susceptances B_w into the single lumped susceptance to be measured. Thus with

$$B = 2B_w \tag{38}$$

and d = 0, (27b) becomes

$$B = 2G_c \tan \frac{1}{2} \beta(s'-s)_{max} \tag{39}$$

where $G_c = 1/R_c$ is the characteristic conductance of the line. For sufficiently small susceptances,

$$B \doteq G_c \beta(s'-s)_{max} \tag{40}$$

The method also may be used to determine the reactive properties of loaded sections of line and variable capacitive tuners.*

The extreme-shift method permits the accurate experimental determination of dielectric constants, permeabilities, and lumped susceptances from measurements of length, viz., $(s'-s)_{max}$, $(s'-s)_{min}$ and d. The accuracy is enhanced by the fact that in measuring the dielectric constant the sample is located at a voltage maximum where its effect is greatest; similarly, when measuring permeability, the sample is located at a current maximum where its effect is again a maximum. Incidentally, the method also may be used to determine the reactive properties of loaded sections of transmission line and of variable capacitive tuners.*

Determination of Losses in Dielectric and Magnetic Materials Using the Maximum-Shift Method.

In the preceding discussion a method is described for determining the real effective dielectric constant $\epsilon_e = \epsilon_o \epsilon_{er}$ and the real permeability $\mu = \mu_o \mu_r$ of a sample of material that could be moved along a transmission line. The conditions (18) that it was convenient to impose

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*R. King, Phil. Mag. Ser. 7, 25, 339 (Feb. 1938).

require this sample (designated as region 1) to have small (but not necessarily zero) attenuation constant α_1 and distortion factor ϕ_{c1}. These quantities are defined as follows for a moderately low-loss line:

$$\frac{\alpha_1}{\beta_1} \doteq \frac{1}{2\omega}\left(\frac{r_1}{\ell_1} + \frac{g_1}{c_1}\right) \quad ; \quad \left(\frac{\alpha_1}{\beta_1}\right)^2 << 1 \tag{41a}$$

$$\phi_{c1} \doteq \frac{1}{2\omega}\left(\frac{r_1}{\ell_1} - \frac{g_1}{c_1}\right) \quad ; \quad \phi_{c1}^2 << 1 \tag{41b}$$

In their usual application r_1 involves only ohmic losses resulting from imperfect conductors and g_1 the ohmic losses of an imperfect dielectric. However, losses in the line may result from time-lags in the polarization response of a dielectric medium with a contribution to the effective conductivity and hence to g_1 or from time-lags in the magnetization response of a magnetic medium with a contribution to the effective resistance r_1. Time lags in the conduction response of a medium involve contributions to the effective dielectric constant as well as to the effective conductivity. All these possible effects are included in the following general formulas for moderately low-loss lines

$$\frac{r}{\omega\ell} = \frac{r^i}{\omega\ell} + \frac{\mu''}{\mu'} \equiv \frac{r^i}{\omega\ell} + h_m \tag{42a}$$

$$\frac{g}{\omega c} = \frac{\sigma_e}{\omega\epsilon_e} = \frac{\sigma' + \omega\epsilon''}{\omega\epsilon' - \sigma''} \equiv h_e \tag{42b}$$

Note that if the conductors are perfect so that the ohmic resistance $r^i = 0$, and the losses in the dielectric medium are not from conduction $\sigma' = \sigma'' = 0$, but exclusively from time lags in polarization and magnetization, (42a) and (42b) reduce to the following symmetrical forms:

$$\frac{r}{\omega\ell} = \frac{\mu''}{\mu'} = h_m \quad ; \quad \frac{g}{\omega c} = \frac{\epsilon''}{\epsilon'} = h_e \tag{43}$$

It is assumed in the following that the imaginary parts of the complex permeability, complex dielectric constant, and complex conductivity

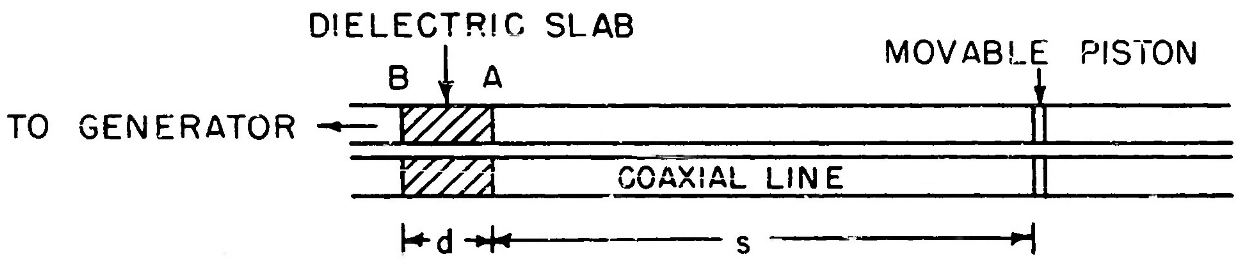

FIG. 5 DIELECTRIC SLAB IN COAXIAL LINE

are small compared with the real parts so that the following formulas are good approximations:

$$\underline{\mu} = \mu' + j\mu'' \doteq \mu + j\mu'' \quad ; \quad \mu = \sqrt{(\mu')^2 + (\mu'')^2} \doteq \mu' \tag{44a}$$

$$\underline{\epsilon} = \epsilon' + j\epsilon'' \doteq \epsilon + j\epsilon'' \quad ; \quad \epsilon = \sqrt{(\epsilon')^2 + (\epsilon'')^2} \doteq \epsilon' \tag{44b}$$

$$\underline{\sigma} = \sigma' + j\sigma'' \doteq \sigma + j\sigma'' \quad ; \quad \sigma = \sqrt{(\sigma')^2 + (\sigma'')^2} \doteq \sigma' \tag{44c}$$

It follows that

$$h_m = \mu''/\mu' = \mu''/\mu \quad ; \quad h_e \equiv \frac{\sigma_e}{\omega\epsilon_e} \doteq \frac{\sigma + \omega\epsilon''}{\omega\epsilon - \sigma''} \tag{45}$$

By adding subscripts 1 and substituting appropriate quantities in (41a,b), the attenuation constant and distortion factor of the dielectric and magnetic ~~medium~~ are

$$\frac{\alpha_1}{\beta_1} \doteq \frac{r^i}{\omega l} + h_m + h_e \;;\; \left(\frac{\alpha_1}{\beta_1}\right)^2 << 1 \tag{46a}$$

$$\phi_{c1} \doteq \frac{r^i}{\omega l} + h_m - h_e \;;\; \phi_{c1}^2 << 1 \tag{46b}$$

For the same line in air (vacuum), the corresponding quantities are

$$\frac{\alpha}{\beta} \doteq \phi_c \doteq \frac{r^i}{\omega l} \;;\; \left(\frac{\alpha_1}{\beta_1}\right)^2 << 1 \tag{47}$$

With these preliminary definitions summarized, attention can be directed to the evaluation of the effective terminal function ρ of the section of line to the right of B (Fig. 5) including the dielectric and magnetic sample and the reactive section of length s.

The admittance looking to the right at A in Fig. 5 is $\underline{Y}_2$ as defined in (3a). The admittance looking to the right at B is $\underline{Y}$ as given in (3b). Since the effect of solid retaining walls (if the material under study is a liquid) is assumed to be purely reactive, there is no contribution to the dissipation, and it is adequate to treat only the

simpler case without walls. This is obtained with $\underline{Y}_w = 0$ in (3a) and (3b). The resulting expressions are

$$\underline{Y}_2 = G_2 + jB_2 = \underline{Y}_c \coth(\underline{\gamma} s + \underline{\theta}'_T) \doteq \underline{Y}_c[(\alpha s \csc^2 \beta s - j \cot \beta s)] \qquad (48)$$

$$\underline{Y} = G + jB = \underline{Y}_{c1}\left[\frac{\underline{Y}_2 \coth \underline{\gamma}_1 d + \underline{Y}_{c1}}{\underline{Y}_2 + \underline{Y}_{c1} \coth \underline{\gamma}_1 d}\right] \qquad (49)$$

The characteristic admittance of the line in the dielectric or magnetic medium is $\underline{Y}_{c1}$, that with the line in air is $\underline{Y}_c$. The two quantities are given by

$$\underline{Y}_c = G_c(1 + j\phi_c) \;;\; G_c = \sqrt{\frac{c}{\ell}} \qquad (50)$$

$$\underline{Y}_{c1} = G_{c1}(1+j\phi_{c1}) \;;\; G_{c1} = \sqrt{\frac{c_1}{\ell_1}} = G_c\sqrt{\frac{\epsilon_{er}}{\mu_r}} \qquad (51)$$

where ϕ_{c1} and ϕ_c are given in (46b) and (47) and $\epsilon_{er} = \epsilon_e/\epsilon_o$ and $\mu_r = \mu/\mu_o$ are the relative values of the real effective dielectric constant and the real permeability.

When the dielectric or magnetic sample is in a position of <u>extreme shift,</u> the susceptance B_2 is the negative of B as shown in (28b). That is

$$B_2 = -B \qquad (52)$$

so that (49) becomes

$$G - jB_2 = \underline{Y}_{c1}\left[\frac{(G_2+jB_2)\coth \underline{\gamma}_1 d + \underline{Y}_{c1}}{G_2 + jB_2 + \underline{Y}_{c1} \coth \underline{\gamma}_1 d}\right] \qquad (53)$$

Let the admittances be normalized by dividing by $\underline{Y}_c$, the characteristic admittance of the air-filled line. As in (21) let

$$\underline{r}_c \equiv \frac{\underline{Y}_{c1}}{\underline{Y}_c} = \frac{R_c(1-j\phi_c)}{R_{c1}(1-j\phi_{c1})} \doteq r_c(1+j\phi_r) \quad , \qquad (54a)$$

where

$$r_c = \frac{R_c}{R_{c1}} = \frac{G_{c1}}{G_c} \;;\; \phi_r = \phi_{c1} - \phi_c = h_m - h_e \;;\; \phi_r^2 \ll 1 \qquad (54b)$$

With (54a,b), (53) becomes

$$g - jb_2 = \underline{r}_c \left[\frac{(g_2+jb_2)\coth \underline{\gamma}_1 d + \underline{r}_c}{g_2+jb_2 + \underline{r}_c \coth \underline{\gamma}_1 d} \right] \tag{55}$$

where

$$(g-jb_2) = (G-jB_2)/\underline{Y}_c \quad ; \quad g_2+jb_2 = (G_2+jB_2)/\underline{Y}_c \tag{56}$$

The real and imaginary parts may be separated using (54a) and (19). For convenience let

$$C_{1r} + jC_{1i} = \underline{r}_c \coth \underline{\gamma}_1 d \doteq r_c(\alpha_1 d \csc^2\beta_1 d + \phi_r \cot\beta_1 d) - jr_c \cot\beta_1 d \tag{57a}$$

where use has been made of (19) and a higher-order term with factor $\alpha_1 d\phi_r$ has been neglected. Also let

$$C_{2r} + jC_{2i} \equiv \underline{r}_c^2 \doteq r_c^2(1 + j2\phi_r) \tag{57b}$$

With this short-hand notation introduced in (55), the following fundamental equations are obtained:

$$b_2^2 + 2b_2C_{1i} - C_{2r} - C_{1r}(g_2-g) + gg_2 = 0 \tag{58a}$$

$$(g-g_2)(b_2+C_{1i}) - 2b_2C_{1r} - C_{2i} = 0 \tag{58b}$$

The conditions (18) imply the following inequality

$$C_{2r} \gg \left| C_{1r}(g-g_2) + gg_2 \right| \tag{59}$$

since when (59) is satisfied, and with (15) and

$$b_2 = -\cot(\beta s + \Phi_T') = -\cot\left(\frac{k\pi}{2} - \Delta\right) = \begin{cases} \cot\Delta; & k \text{ even} \\ -\tan\Delta; & k \text{ odd} \end{cases} \tag{60}$$

(58a) reduces exactly to the fundamental equations (14a,b) for the conditions of extreme shift. By combining (14a,b) with (60) the following alternative expressions are obtained for b_2:

$$b_2 = \begin{cases} r_c \cot \frac{1}{2}\beta_1 d & ; \quad k \text{ even} \\ \\ -r_c \tan \frac{1}{2}\beta_1 d & ; \quad k \text{ odd} \end{cases} \tag{61}$$

The remaining equation (58b) is to be used to determine g and from it ρ. Since the section of line to the right of the dielectric slab (Fig. 5) is essentially reactive, it may be assumed that g_2 is negligible compared with g. Hence

$$g - g_2 \doteq g \doteq \frac{2b_2 C_{1r} + C_{2i}}{b_2 + C_{1i}} \tag{62}$$

With (57a,b) and (61), (62) may be expressed as follows:

$$g \doteq 2r_c \left[\alpha_1 d \csc^2 \beta_1 d + \phi_r \cot \beta_1 d) \cot \frac{1}{2}\beta_1 d + \phi_r \right] \sin \beta_1 d \tag{63a}$$

k even

$$g \doteq 2r_c [(\alpha_1 d \csc^2 \beta_1 d + \phi_r \cot \beta_1 d) \tan \frac{1}{2}\beta_1 d - \phi_r] \sin \beta_1 d \tag{63b}$$

k odd

Use has been made of the identities

$$\tan \frac{1}{2} x = \csc x - \cot x \tag{64a}$$

$$\cot \frac{1}{2} x = \csc x + \cot x \tag{64b}$$

The terminal attenuation function ρ of a moderately low-loss line may be determined from

$$\rho = \frac{1}{2} \tanh^{-1} \frac{2g}{1+b^2+g^2} \doteq \frac{g}{1+b^2} \tag{65}$$

The substitution of (23a) or (23b) in (25) together with the appropriate formula from (20) leads to

$$\rho_i = \frac{2r_c\left[\frac{\alpha_1 d}{2}\sec^2\frac{1}{2}\beta_1 d + \phi_r \tan\frac{1}{2}\beta_1 d\right]}{r_c^2 + \tan^2\frac{1}{2}\beta_1 d} \quad ; \quad k \text{ even} \qquad (66a)$$

$$\rho_v = \frac{2r_c\left[\frac{\alpha_1 d}{2}\sec^2\frac{1}{2}\beta_1 d - \phi_r \tan\frac{1}{2}\beta_1 d\right]}{1 + r_c^2\tan^2\frac{1}{2}\beta_1 d} \quad ; \quad k \text{ odd} \qquad (66b)$$

(The subscripts i and v indicate ρ respectively, with current or voltage maximum at the center of the slab.) In deriving (26a,b) use has been made of (24a) to express all arguments as $\frac{1}{2}\beta_1 d$.

Since with k even the dielectric sample has a current maximum at its center $(\beta_1 d < \pi)$, with k odd a voltage maximum, the values of ρ in (46a) and (46b) are twice the values obtained, respectively, by placing a slab of dielectric of thickness d/2 at an ideal short-circuited end and an ideal open end.

Since the circuit is always adjusted to resonance in determining the extreme shift, it is convenient to determine ρ_i and ρ_v using the resonance-curve method. Once these two quantities are known α_1 and ϕ_r may be evaluated from (26a) and (26b) and from these h_m and h_e using (46a,b) with (54b). It is assumed that the constants of the line in air are known, as well as R_{ci} and β_1 which involve ϵ_r and μ_r.

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